Eur. Phys. J. C 52, 75–82 (2007)
DOI 10.1140/epjc/s10052-007-0351-2

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

QED processes in peripheral kinematics at polarized photon–photon and photon–electron colliders

S. Bakmaev[1,a], E.A. Kuraev[1], E. Bartoš[2], M. Galinskii[3]

[1] Joint Institute for Nuclear Research, Joliot Curie 6, Dubna, 141980, Russia
[2] Institute of Physics SAS, Bratislava, 845 11, Slovakia
[3] Institute of Physics BAS, Minsk, 220072, Belarus

Received: 18 April 2007 / Revised version: 31 May 2007 /
Published online: 13 July 2007 –

Abstract. The calibration QED process cross sections for experiments on planned electron–photon and photon–photon colliders for detecting small angle scattered particles are calculated. These processes describe the creation of two jets moving sufficiently close to the beam axis directions. The jets containing two and three particles including charged leptons, photons, and pseudoscalar mesons are considered explicitly. Considering the pair production subprocesses we take into account both bremsstrahlung and double photon mechanisms. The obtained results are suitable for further numerical calculations.

PACS. 13.40.-f; 13.88.+e

1 Introduction

QED processes of the type $2 \to 3, 4, 5, 6$ at colliders of high energies have attracted both theoretical and experimental attention during the last four decades. Accelerators with high-energy colliding e^+e^-, γe, $\gamma\gamma$ and $\mu^+\mu^-$ beams have been designed and widely used to study the fundamental interactions [1–4]. Some processes of quantum electrodynamics (QED) can play an important role at these colliders, especially those inelastic processes whose cross section does not drop with increasing energy. The planned colliders will be able to work with polarized particles, so these QED processes are required to be described in more detail, including the calculation of cross sections with definite helicities of the initial particles – leptons ($l = e$ or μ) and photons γ. These reactions have the form of a two-jet process with the exchange of a virtual photon γ^* in the t-channel (see Fig. 1).

Much attention was paid in the literature to the calculation of helicity amplitudes of QED processes at high-energy colliders (see [5] and references therein). Keeping in mind the physical programs at planned $\gamma\gamma$ and lepton γ colliders, precise knowledge of a set of calibration and monitoring processes is needed. This refers to calibration processes as well as to QED processes with sufficiently large cross sections and clear signatures for detection. Rather rich physics can be investigated in peripheral processes such as heavy leptons and mesons (scalar and pseudoscalar) creation, where the relevant QED monitoring processes must be measured.

Let us recall the general features of peripheral processes, namely, the important fact of their nondecreasing cross sections in the limit of high total energies $\sqrt{s}$ in the center of mass frame of the initial particles. The possibility of measuring the jets containing two or three particles can be relevant. This is a motivation of our paper.

It is organized in the following way.

In Sect. 2, the kinematics of peripheral processes is briefly described.

In Sect. 3, the impact factors describing the conversion of the initial photon to a pair of charged particles (fermions or spinless mesons) with or without an additional hard photon are calculated.

In Sects. 4–6 a similar calculation is made for the initial polarized electron and photon; in particular, such subprocesses as the single and double Compton process and the processes of pair creation are considered.

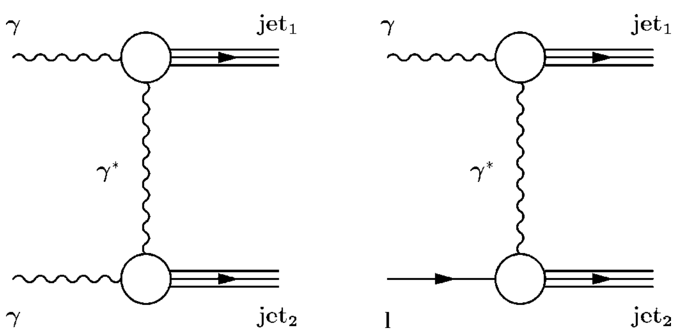

Fig. 1. The processes $\gamma\gamma$, γl $(l = e, \mu)$ with the exchange of a virtual photon γ^* in the t-channel

[a] email: bakmaev@theor.jinr.ru

As the helicity amplitudes for subprocesses of type $2 \to 3$ have in general a complicated form, we do not write explicit expressions for the corresponding cross sections, indicating only the strategy to obtain them.

2 Kinematics

Throughout the paper it is implied that the energy fractions of a jet component are positive quantities of the order of unity (the sum of energy fractions of each jet is unity) and the values of the components of their 3-momenta transversal to the beam direction are much larger than their rest masses. So we neglect the mass of jet particles.

The corresponding amplitudes include a large amount of Feynman diagrams (FD). Fortunately, in the high-energy limit the number of essential FD contributing to the "leading" approximation greatly reduces. The method used permits one to estimate the uncertainty caused by "nonleading" contributions that have the following magnitudes of order:

$$\frac{m^2}{s_1}\,, \quad \frac{s_1}{s}\,, \quad \frac{s_2}{s}\,, \quad \frac{\alpha}{\pi}\ln\frac{s}{m^2}\,, \tag{1}$$

where $s_{1,2}$ are the jet invariant mass squares to be compared with the terms of the order of unity. The last term in (1) is caused by the absence of radiative corrections in our analysis. The angles θ_i of particle emission with respect to the corresponding direction of motion of the projectile is assumed to be of the order of (see Fig. 2)

$$\frac{m_i}{\sqrt{s}} \ll \theta_i \sim \frac{\sqrt{s_i}}{\sqrt{s}} \ll 1\,, \tag{2}$$

where m_i is the typical mass of the jet particle.

In this approach we consider the initial particles (having the 4-momenta p_1, p_2) as massless and use the Sudakov parameterization of the 4-momenta of any particle of the problem:

$$q_i = \alpha_i p_2 + \beta_i p_1 + q_{i\perp}\,, \tag{3}$$
$$q_{i\perp} p_{1,2} = 0\,, \quad q_{i\perp}^2 = -\mathbf{q}_i^2 < 0\,.$$

The Sudakov parameters β_i are quantities of the order of unity for the momenta of the particles belonging to jet1 and obeying the conservation law $\sum_{\mathrm{jet1}} \beta_i = 1$, whereas the components of the jet1 particle momenta along the 4-momentum p_2 are small positive numbers that can be determined from the on mass shell conditions of the jet1 particles, $q_i^2 = s\alpha_i\beta_i - \mathbf{q}_i{}^2 = 0$, $\alpha_i = \mathbf{q}_i{}^2/(s\beta_i) \ll 1$.

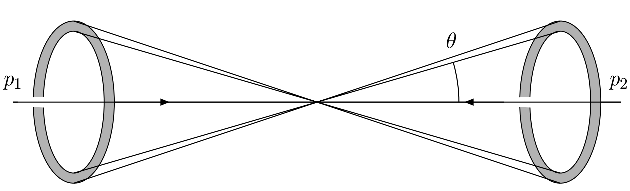

Fig. 2. The scheme of collision of initial beams with detection of two jets moving in the cones within the angles θ

The same is valid for the 4-momenta of the particles belonging to jet2, namely, $\alpha_j \sim 1$, $\sum_{\mathrm{jet2}} \alpha_j = 1$, $\beta_j = \mathbf{q}_j{}^2/(s\alpha_j) \ll 1$.

Among the large amount of FD describing the process in the lowest (Born) order of perturbation theory (PT) (tree approximation), only those survive (i.e., give contributions to the cross section that do not decrease with increasing s) that have a photonic t-channel one-particle state.

It is known [6] that the matrix elements of the peripheral processes have a factorized form and the cross section can be written in terms of the so-called impact factors, each of which describe the subprocess of the interaction of the internal virtual photon with one of the initial particles to produce a jet moving in a direction close to this projectile momentum. So the problem can be formulated in terms of the computation of impact factors. For processes with initial photons with a definite state of polarization described in terms of Stoke's parameters, we construct the relevant chiral matrices from bilinear combinations of chiral amplitudes. The last step consists in the construction of differential cross sections.

The matrix element that corresponds to the main ("leading") contribution to the cross section has the form

$$M = \mathrm{i} J_1^\mu \frac{g_{\mu\nu}}{q^2} J_2^\nu\,, \tag{4}$$

where J_1^μ and J_2^ν are the currents of the upper (associated with jet1) and lower blocks of the relevant Feynman diagram, respectively, and $g_{\mu\nu}$ is the metric tensor. The current J_1^μ describes the scattering of an incoming particle of momentum p_1 with a virtual photon and the subsequent transition to the first jet (and similar for J_2^ν). The matrix elements (4) can be written in the form (see the appendices in [6])

$$M = 2\mathrm{i}\frac{s}{q^2} I_1 I_2\,, \tag{5}$$
$$I_1 = \frac{1}{s} J_1^\mu p_{2\mu}\,, \quad I_2 = \frac{1}{s} J_2^\nu p_{1\nu}\,.$$

In fact, this follows from the Gribov representation of the metric tensor,

$$g^{\mu\nu} = \frac{2}{s}(p_2^\mu p_1^\nu + p_2^\nu p_1^\mu) + g_\perp^{\mu\nu} \approx \frac{2}{s} p_2^\mu p_1^\nu\,. \tag{6}$$

The invariant mass squares of jets can also be expressed in terms of the Sudakov parameters of the exchanged photon,

$$\begin{aligned} q &= \alpha p_2 + \beta p_1 + q_\perp\,,\\ (q+p_1)^2 &= s_1 = -\mathbf{q}^{\,2} + s\alpha\,,\\ (-q+p_2)^2 &= s_2 = -\mathbf{q}^{\,2} - s\beta\,,\\ q^2 &= s\alpha\beta - \mathbf{q}^2 \approx -\mathbf{q}^2\,. \end{aligned} \tag{7}$$

Here and below, by the symbol $\approx$ we mean the equation with neglect of the terms that do not contribute to the limit $s \to \infty$.

The singularity of the matrix element (5) at $\mathbf{q}=0$ is fictitious (excluding elastic scattering). In fact, one can see that it cancels due to the current conservation:

$$q_\mu J_1^\mu \approx (\alpha p_2 + q_\perp)_\mu J_1^\mu = 0\,, \quad p_{2\mu} J_1^\mu = \frac{s}{s\alpha}\mathbf{q}\mathbf{J}_1\,, \quad (8)$$

$$q_\nu J_2^\nu \approx (\beta p_1 + q_\perp)_\nu J_2^\nu = 0\,, \quad p_{1\nu} J_2^\nu = \frac{s}{s\beta}\mathbf{q}\mathbf{J}_2\,. \quad (9)$$

We arrive at the modified form of the matrix element of a peripheral process:

$$\begin{aligned} &M(a(p_1,\eta_1)+b(p_2,\eta_2)) \to \mathrm{jet}_{1\lambda_1}+\mathrm{jet}_{2\lambda_2} \\ &= \mathrm{i}(4\pi\alpha)^{\frac{n_1+n_2}{2}}\frac{2s}{\mathbf{q}^2} m_{1\lambda_1}^{\eta_1} m_{2\lambda_2}^{\eta_2}\,, \quad (10) \\ &\eta_i, \lambda_i = \pm 1; \quad \eta_{1,2} \geq 2, \end{aligned}$$

where η_i, describe the polarization states of the projectile $i=a,b$; λ_i describes the polarization states of the participants of its initiated jet. The numbers of QED vertices in the upper and lower blocks of FD (see Fig. 1) are denoted by $n_{1,2}$.

We give here two alternative forms for the matrix elements $m_{1,2}$ of the subprocesses $\gamma^*(q)+a(p_1,\eta_1)\to \mathrm{jet}_{1(\lambda_1)}$ and $\gamma^*(q)+b(p_2,\eta_2)\to \mathrm{jet}_{2(\lambda_2)}$:

$$m_{1\lambda_1}^{\eta_1} = \frac{\mathbf{q}\mathbf{J}_{1\lambda_1}^{\eta_1}}{s_1+\mathbf{q}^2} \quad (11)$$

$$m_{1\lambda_1}^{\eta_1} = \frac{1}{s} p_{2\mu} J_{1\lambda_1}^{\eta_1\mu}\,, \quad (12)$$

and similar expressions hold for the lower block. We use the second representation (12). The form (11) can be used as a check of the validity of gauge invariance, namely, turning the matrix elements to zero in the limit $\mathbf{q}\to 0$.

A remarkable feature of the peripheral processes is that their differential cross sections do not depend on the total center of mass energy $\sqrt{s}$. To see this property, let us first rearrange the phase volume $\mathrm{d}\Phi$ of the final two-jet kinematics state to a more convenient form:

$$\begin{aligned} \mathrm{d}\Phi &= (2\pi)^4\delta^4\left(p_1+p_2-\sum_i p_i^{(1)} - \sum_j p_j^{(2)}\right) \\ &\quad\times \mathrm{d}F^{(1)}\,\mathrm{d}F^{(2)} \\ &= (2\pi)^4\,\mathrm{d}^4 q\,\delta^4_{(1)}\delta^4_{(2)}\,\mathrm{d}F^{(1)}\,\mathrm{d}F^{(2)}\,, \quad (13) \\ \delta^4_{(1)} &= \delta^4\left(p_1+q-\sum_i p_i^{(1)}\right), \\ \delta^4_{(2)} &= \delta^4\left(p_2-q-\sum_j p_j^{(2)}\right), \\ \mathrm{d}F_{(1,2)} &= \prod_i \frac{\mathrm{d}^3 p_i^{(1,2)}}{2\varepsilon_i^{(1,2)}(2\pi)^3}\,. \end{aligned}$$

Using Sudakov's parameterization for the transferred 4-momentum q phase volume,

$$\mathrm{d}^4 q = \frac{s}{2}\,\mathrm{d}\alpha\,\mathrm{d}\beta\,\mathrm{d}^2 q_\perp = \frac{1}{2s}\,\mathrm{d}s_1\,\mathrm{d}s_2\,\mathrm{d}^2 q_\perp\,, \quad (14)$$

with $s_{1,2}$ being the invariant mass squares of the jets, we put the phase volume in the factorized form

$$\mathrm{d}\Phi = \frac{(2\pi)^4}{2s}\,\mathrm{d}^2 q_\perp\,\mathrm{d}s_1\,\mathrm{d}F^{(1)}\delta^4_{(1)}\,\mathrm{d}s_2\,\mathrm{d}F^{(2)}\delta^4_{(2)}\,. \quad (15)$$

Using the modified form of the matrix element and the phase volume for the peripheral process cross section in the case of polarized initial particles (photons or electrons), we have

$$\mathrm{d}\sigma^{\eta_1\eta_2} = \frac{\alpha^{n_1+n_2}\pi^2(4\pi)^{2+n_1+n_2}\,\mathrm{d}^2 q_\perp}{(\mathbf{q}^2)^2}\Phi_1^{\eta_1}(\mathbf{q})\Phi_2^{\eta_2}(\mathbf{q})\,, \quad (16)$$

with the impact factors $\Phi_i^{\eta_i}$ in the form

$$\Phi_i^{\eta_i}(\mathbf{q}) = \int \mathrm{d}s_i \sum_{\lambda_j} |m_{i\lambda_j}^{\eta_i}|^2\,\mathrm{d}F_i\delta^4_{(i)}\,, \quad i=1,2\,. \quad (17)$$

The matrix elements with the definite chiral states of all particles $m_{i(\lambda)}^{\eta_i}$, where the subscript (λ) denotes the set of chiral parameters of the final state, are calculated and listed below.

In the case of initial polarized photons the description in terms of Stoke's parameters $\xi_{1,2,3}$, $\xi_1^2+\xi_2^2+\xi_3^2\leq 1$ is commonly used. The matrix element squared on the r.h.s. of (17) must be replaced by [7]

$$\begin{aligned} T_\gamma &= \mathrm{Sp}(\mathcal{M}\rho) \\ &= \frac{1}{2}\mathrm{Sp}\begin{pmatrix} m^{++} & m^{+-} \\ m^{-+} & m^{--}\end{pmatrix}\begin{pmatrix} 1+\xi_2 & \mathrm{i}\xi_1-\xi_3 \\ -\mathrm{i}\xi_1-\xi_3 & 1-\xi_2\end{pmatrix}\,, \end{aligned} \quad (18)$$

with the spin matrix $\mathcal{M}$ elements

$$\begin{aligned} m^{++} &= \sum_\lambda \left|m^+_{(\lambda)}\right|^2\,, \quad m^{+-} = \sum_\lambda m^+_{(\lambda)}\left(m^-_{(\lambda)}\right)^*\,, \\ m^{--} &= \sum_\lambda \left|m^-_{(\lambda)}\right|^2\,, \quad m^{-+} = \left(m^{+-}\right)^*\,. \end{aligned} \quad (19)$$

We choose $\lambda=+1$ for the initial fermion

$$T_e = \sum_\lambda \left|m^+_\lambda\right|^2\,. \quad (20)$$

The cross sections $\mathrm{d}\sigma_{n_1,n_2}$ of the process of type $2\to n_1+n_2$ with production of two jets,

$$\begin{aligned} a(p_1,\eta_1)+b(p_2,\eta_2) &\to a_1(r_1\lambda_1)+\dots+a_{n_1}(r_{n_1},\lambda_{n_1}) \\ &\quad + b_1(q_1,\sigma_1)+\dots+b_{n_2}(q_{n_2},\sigma_{n_2})\,, \end{aligned} \quad (21)$$

where the energy fractions are denoted $x_1,\dots x_{n_1}$, $\sum x_i = 1$, and the transversal components of the momenta are $\mathbf{r}_1,\dots \mathbf{r}_{n_1}$, and $\sum \mathbf{r}_i = \mathbf{q}$ of jet a and similar for the quantities $y_i, \mathbf{q}_i$, $\sum y_i = 1$, $\sum \mathbf{q}_i = -\mathbf{q}$, for the other jet b, have the form

$$\mathrm{d}\sigma_{22} = \frac{\alpha^4}{2^2\pi^4} T_2^{(1)} T_2^{(2)} \frac{\mathrm{d}^2 q}{(\mathbf{q}^2)^2} \mathrm{d}^2 r_1 \, \mathrm{d}^2 q_1 \frac{\mathrm{d}x_1 \, \mathrm{d}y_1}{x_1 x_2 y_1 y_2}, \quad (22)$$

$$\mathrm{d}\sigma_{23} = \frac{\alpha^5}{2^4\pi^6} T_2^{(1)} T_3^{(2)} \frac{\mathrm{d}^2 q}{(\mathbf{q}^2)^2} \mathrm{d}^2 r_1 \, \mathrm{d}^2 q_1 \, \mathrm{d}^2 q_2 \frac{\mathrm{d}x_1 \, \mathrm{d}y_1 \, \mathrm{d}y_2}{x_1 x_2 y_1 y_2 y_3}, \quad (23)$$

$$\mathrm{d}\sigma_{33} = \frac{\alpha^6}{2^6\pi^8} T_3^{(1)} T_3^{(2)} \frac{\mathrm{d}^2 q}{(\mathbf{q}^2)^2} \mathrm{d}^2 q_1 \, \mathrm{d}^2 q_2 \, \mathrm{d}^2 r_1 \, \mathrm{d}^2 r_2 \times \frac{\mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}y_1 \, \mathrm{d}y_2}{x_1 x_2 x_3 y_1 y_2 y_3}. \quad (24)$$

3 Subprocesses $\gamma^*\gamma \to e^+e^-$, $\pi^+\pi^-$

Let us consider first the contribution to the impact factor of the photon from the lepton pair production subprocess

$$\gamma(k_1, \eta) + \gamma^*(q) \to e^-(q_-, \lambda) + e^+(q_+, -\lambda).$$

The matrix element of the subprocess has the form (we suppress the factor $4\pi\alpha$)

$$m_{1\lambda}^{\eta\mu} = -\bar{u}_\lambda(q_-) \left[\hat{\varepsilon}^\eta \frac{\hat{q}_- - \hat{k}_1}{\kappa_{1-}} \gamma^\mu + \gamma^\mu \frac{-\hat{q}_+ + \hat{k}_1}{\kappa_{1+}} \hat{\varepsilon}^\eta \right] v_\lambda(q_+),$$
$$\bar{u}_\lambda = \bar{u}\omega_{-\lambda}, \quad v_\lambda = \omega_{-\lambda} v. \quad (25)$$

We imply all the particles to be massless. A definite chiral state initial photon polarization vector has the form [8]

$$\hat{\varepsilon}_1^\lambda = N_1 \left[\hat{q}_- \hat{q}_+ \hat{k}_1 \omega_{-\lambda} - \hat{k}_1 \hat{q}_- \hat{q}_+ \omega_\lambda \right], \quad (26)$$

where

$$N_1^2 = \frac{2}{s_1 \kappa_+ \kappa_-}, \quad s_1 = 2q_+ q_-, \quad \kappa_{1\pm} = 2k_1 q_\pm. \quad (27)$$

The chiral amplitudes $m_\lambda^\eta = (1/s) m_{1\lambda}^{\eta\mu} p_{2\mu}$ have the form

$$m_{1+}^+ = -\frac{N_1}{s} \bar{u} \hat{q}_+ \hat{q} \hat{p}_2 \omega_+ v,$$
$$m_{1-}^+ = -\frac{N_1}{s} \bar{u} \hat{p}_2 \hat{q} \hat{q}_- \omega_- v,$$
$$m_{1-}^- = -\frac{N_1}{s} \bar{u} \hat{q}_+ \hat{q} \hat{p}_2 \omega_- v,$$
$$m_{1+}^- = -\frac{N_1}{s} \bar{u} \hat{p}_2 \hat{q} \hat{q}_- \omega_+ v. \quad (28)$$

The elements of the spin matrix $\mathcal{M}$ in the case of lepton pair production are

$$m_{e^+e^-}^{++} = m_{e^+e^-}^{--} = \frac{2\mathbf{q}^2}{\mathbf{q}_+^2 \mathbf{q}_-^2} x_+ x_- (x_+^2 + x_-^2), \quad (29)$$
$$m_{e^+e^-}^{+-} = (m_{e^+e^-}^{-+})^* = -\frac{4\mathbf{q}^2}{\mathbf{q}_+^2 \mathbf{q}_-^2} (x_+ x_-)^2 \mathrm{e}^{2\mathrm{i}\theta},$$

$x_\pm$ are the energy fractions carried out by pair components, $x_+ + x_- = 1$, and θ is the angle between two Euclidean vectors $\mathbf{q} = \mathbf{q}_- + \mathbf{q}_+$ and $\mathbf{Q} = x_+ \mathbf{q}_- - x_- \mathbf{q}_+$.

In the case of charged pion pair production

$$\gamma(p_1, e_1^\eta) + \gamma^*(q) \to \pi^+(q_+) + \pi^-(q_-), \quad (30)$$

we have

$$m^\eta = \frac{1}{s} \varepsilon_{1\nu}^\eta p_2^\mu m_\mu^\nu = \frac{x_+}{p_1 q_-} \varepsilon_1^\eta q_- + \frac{x_-}{p_1 q_+} \varepsilon_1^\eta q_+ - \frac{2}{s} (\varepsilon_1^\eta p_2). \quad (31)$$

Using the photon polarization vector written as

$$\varepsilon_{1\mu}^\eta = N_1 \left[(q_+ p_1) q_{-\mu} - (q_- p_1) q_{+\mu} + \mathrm{i}\eta \varepsilon_{\mu\alpha\beta\gamma} q_-^\alpha q_+^\beta p_1^\gamma \right] \quad (32)$$

we obtain the chiral amplitude of the pion pair production process (we define $(p_1 p_2 q_- q_+) = \epsilon_{\alpha\beta\gamma\delta} p_1^\alpha p_2^\beta q_-^\gamma q_+^\delta = (s/2)[\mathbf{q}_- \mathbf{q}_+]_z$):

$$m^\eta = -N_1 (\mathbf{Q}\mathbf{q} + \mathrm{i}\eta [\mathbf{Q}, \mathbf{q}]_z) = -N_1 |\mathbf{q}| \, |\mathbf{Q}| \mathrm{e}^{\mathrm{i}\eta\theta}, \quad \theta = \widehat{\mathbf{q}\mathbf{Q}}. \quad (33)$$

where we imply the z axis direction to be along the photon 3-vector, and we use the relation $[\mathbf{q}_-, \mathbf{q}_+]_z = [\mathbf{Q}, \mathbf{q}]_z$. For the pion chiral matrix we have

$$m_{\pi^+\pi^-}^{++} = m_{\pi^+\pi^-}^{--} = \frac{2\mathbf{q}^2}{\mathbf{q}_+^2 \mathbf{q}_-^2} (x_+ x_-)^2,$$
$$m_{\pi^+\pi^-}^{+-} = (m_{\pi^+\pi^-}^{-+})^* = \frac{2\mathbf{q}^2}{\mathbf{q}_+^2 \mathbf{q}_-^2} (x_+ x_-)^2 \mathrm{e}^{2\mathrm{i}\theta}. \quad (34)$$

For the two-pair production process

$$\gamma_1(p_1, \boldsymbol{\xi}_1) + \gamma_2(p_2, \boldsymbol{\xi}_2) \to a(q_-) + \bar{a}(q_+) + b(p_-) + \bar{b}(p_+),$$
$$q_\pm = \alpha_\pm p_2 + x_\pm p_1 + q_{\pm\perp}, \quad p_\pm = y_\pm p_2 + \beta_\pm p_1 + p_{\pm\perp}, \quad (35)$$

the differential cross section (assuming that the pair $a\bar{a}$ moves along the 1 direction of the photon, and the pair $b\bar{b}$ moves along the 2 direction) has the form (22) with

$$T^{(1)} = \frac{\mathbf{q}^2}{\mathbf{q}_+^2 \mathbf{q}_-^2} (x_+ x_-)^2 [1 - \xi_3 \cos(2\theta) + \xi_1 \sin(2\theta)] \quad \text{for} \quad \pi^+, \pi^-, \quad (36)$$

$$T^{(1)} = \frac{\mathbf{q}^2}{\mathbf{q}_+^2 \mathbf{q}_-^2} (x_+ x_-) \times \left\{ x_+^2 + x_-^2 + 2x_+ x_- [\xi_3 \cos(2\theta) + \xi_1 \sin(2\theta)] \right\} \quad \text{for} \quad e^+, e^-, \quad (37)$$

and a similar expression holds for $T^{(2)}$[1]. We recall that the formulae obtained are valid at a large transverse component of the jet particles, compared to the masses of the particles,

$$\mathbf{q}_-^2 \sim \mathbf{q}_+^2 \sim \mathbf{p}_+^2 \sim \mathbf{p}_-^2 \gg m^2,$$
$$\mathbf{q}_+ = \mathbf{q} - \mathbf{q}_-, \quad \mathbf{p}_+ = -\mathbf{q} - \mathbf{p}_-, \quad (38)$$

[1] In [9] (37) contains a misprint in the sign of $\xi_3^{(1,2)}$.

and finite energy fractions $x_\pm \sim y_\pm \sim 1$, which correspond to the emission angles of jet particles $\theta_i = |\mathbf{q}_i|/(x_i\varepsilon) \gg m/\varepsilon$ that are considerably larger than the mass to energy ratio.

4 Subprocesses $\gamma^*\gamma \to e^+e^-\gamma, \pi^+\pi^-\gamma$

Here and below for subprocesses of type $2 \to 3$ we restrict ourselves to calculation of the chiral amplitudes and checking their gauge invariance properties.

The subprocess

$$\gamma(k,\lambda) + \gamma^*(q) \to e^+(q_+, -\lambda_-) + e^-(q_-, \lambda_-) + \gamma(k_1, \lambda_1)$$

is described by six FD. A standard calculation of the chiral amplitudes $m^\lambda_{\lambda_1\lambda_-}$ leads to

$$\begin{aligned} m^+_{++} &= -\frac{s_1 N N_1}{s}\bar{u}(q_-)\hat{q}_+\hat{q}\hat{p}_2\omega_+ v(q_+) = (m^-_{--})^*\,, \\ m^+_{+-} &= -\frac{s_1 N N_1}{s}\bar{u}(q_-)\hat{p}_2\hat{q}\hat{q}_-\omega_- v(q_+) = (m^-_{-+})^*\,, \\ m^+_{-+} &= \frac{N N_1}{s}\bar{u}(q_-)A^+_{-+}\omega_+ v(q_+) = (m^-_{+-})^*\,, \\ m^+_{--} &= \frac{N N_1}{s}\bar{u}(q_-)A^+_{--}\omega_- v(q_+) = (m^-_{++})^*\,, \end{aligned} \tag{39}$$

with $A^+_{--}(k,k_1) = A^+_{-+}(-k_1,-k)$

$$\begin{aligned} N^2 &= \frac{2}{s_1\kappa_-\kappa_+}, \quad N_1^2 = \frac{2}{s_1\kappa_{1+}\kappa_{1-}}, \\ s_1 &= 2q_+q_-, \quad \kappa_\pm = 2kq_\pm, \quad \kappa_{1\pm} = 2k_1q_\pm\,, \end{aligned} \tag{40}$$

and a rather cumbersome expression for A^+_{-+}:

$$\begin{aligned} A^+_{-+} = &\frac{s_1}{(q_+-q)^2}\hat{k}\hat{q}_+\hat{k}_1(-\hat{q}_+ + \hat{q})\hat{p}_2 \\ &- \hat{q}_+(\hat{q}_- - \hat{k})\hat{p}_2(\hat{q}_+ + \hat{k}_1)\hat{q}_- \\ &- \frac{s_1}{(q_- - q)^2}\hat{p}_2(\hat{q}_- - \hat{q})\hat{k}\hat{q}_-\hat{k}_1\,. \end{aligned} \tag{41}$$

Substituting

$$\hat{p}_2 \approx \frac{1}{\alpha}(\hat{q} - \hat{q}_\perp) = \frac{s}{s\alpha}\left[\hat{q}_+ + \hat{k}_1 + (\hat{q}_- - \hat{k}) - \hat{q}_\perp\right]\,,$$

in the second term of the r.h.s. of (41) we have

$$\begin{aligned} A^+_{-+} = &-ss_1\kappa_{1+}\left[\frac{x_+}{(q_+-q)^2} + \frac{1}{s\alpha}\right]\hat{k} \\ &- ss_1\kappa_-\left[\frac{x_-}{(q_- - q)^2} + \frac{1}{s\alpha}\right]\hat{k}_1 \\ &+ \frac{s_1}{(q_+-q)^2}\hat{k}\hat{q}_+\hat{k}_1\hat{q}_\perp\hat{p}_2 + \frac{s_1}{(q_- - q)^2}\hat{p}_2\hat{q}_\perp\hat{k}\hat{q}_-\hat{k}_1 \\ &+ \frac{s}{s\alpha}\hat{q}_+(\hat{q}_- - \hat{k})\hat{q}_\perp(\hat{q}_+ + \hat{k}_1)\hat{q}_-\,, \end{aligned} \tag{42}$$

with

$$\begin{aligned} (q_\pm - q)^2 &= -\mathbf{q}^2 + 2\mathbf{q}\mathbf{q}_\pm - s\alpha x_\pm\,, \\ s\alpha &= \frac{\mathbf{k}_1^2}{x_1} + \frac{\mathbf{q}_-^2}{x_-} + \frac{\mathbf{q}_+^2}{x_+}\,, \\ x_1 + x_- + x_+ &= 1, \quad \kappa_\pm = \frac{\mathbf{q}_\pm^2}{x_\pm}, \\ \kappa_{1\pm} &= \frac{1}{x_1x_\pm}(x_1\mathbf{q}_\pm - x_\pm\mathbf{k}_1)^2\,. \end{aligned} \tag{43}$$

The gauge property that the chiral amplitudes must vanish as $\mathbf{q} \to 0$ can be seen explicitly.

A further procedure of constructing the chiral matrix is straightforward and can be performed in terms of simple traces. We will not touch upon this here.

Consider the subprocess

$$\gamma(k,\lambda) + \gamma^*(q) \to \pi^+(q_+) + \pi^-(q_-) + \gamma(k_1,\lambda_1)\,.$$

There are 12 FD describing a rather cumbersome expression for the matrix element. It can considerably be simplified on using the modified expressions for the photon polarization vectors in the form [11, 12]

$$\varepsilon^\lambda_\mu(k) = \frac{N}{2}Sp\gamma_\mu\hat{q}_-\hat{q}_+\hat{k}\omega_\lambda\,, \quad \varepsilon^{\lambda_1}_\mu(k_1) = \frac{N_1}{2}Sp\gamma_\mu\hat{q}_-\hat{q}_+\hat{k}_1\omega_\lambda\,, \tag{44}$$

with the same expressions for N, N_1 as in the case of the $\gamma\gamma^* \to e^+e^-\gamma$ subprocess. Polarization vectors chosen in such a form satisfy the Lorentz condition $\varepsilon(k)k = 0$, $\varepsilon(k_1)k_1 = 0$ and the gauge condition $\varepsilon(k)q_- = \varepsilon(k_1)q_- = 0$.

The matrix element has the form (we lost Bose symmetry at this stage)

$$\begin{aligned} m^\lambda_{\lambda_1} &= \frac{1}{s}p_2^\rho\varepsilon^\mu(k)\varepsilon_1^{*\sigma}(k_1)O_{\rho\mu\sigma} \\ &= \frac{4x_-}{(q_- - q)^2}\left[\frac{(\varepsilon_1 q_+)(\varepsilon q)}{\kappa_{1+}} - \frac{(\varepsilon_1 q)(\varepsilon q_+)}{\kappa_+}\right] \\ &\quad + \frac{4(\varepsilon p_2)(\varepsilon_1 q_+)}{s\kappa_{1+}} - \frac{4(\varepsilon_1 p_2)(\varepsilon q_+)}{s\kappa_+} \\ &\quad + (\varepsilon\varepsilon_1)\left[\frac{x_+}{(q_+-q)^2} - \frac{x_-}{(q_- - q)^2}\right]\,, \end{aligned} \tag{45}$$

where we imply $\varepsilon = \varepsilon^\lambda$, $\varepsilon_1 = \varepsilon_1^{\lambda_1}$ and $x_\pm = 2p_2q_\pm/s$, $x_1 = 2p_2k_1/s$, where $x_+ + x_- + x_1 = 1$.

For $\lambda_1 = \lambda$ we have

$$m^\lambda_\lambda = s_1NN_1[A_1 + \mathrm{i}\lambda B_1]\,, \quad A_1 = -\mathbf{Q}\mathbf{q}, \quad B_1 = [\mathbf{Q}\mathbf{q}]_z\,. \tag{46}$$

In the case of opposite chiralities we have

$$m^\lambda_{-\lambda} = s_1NN_1[A + \mathrm{i}\lambda B],$$

$$A = -\mathbf{Q}\mathbf{q} + \frac{1}{2x_1 x_- x_+}\left[\mathbf{Q}^2\mathbf{k}_1^2 - \mathbf{q}_-^2(x_1\mathbf{q}_+ - x_+\mathbf{k}_1)^2 - \mathbf{q}_+^2(x_1\mathbf{q}_- - x_-\mathbf{k}_1)^2\right]\left(\frac{x_+}{(q_+-q)^2} - \frac{x_-}{(q_--q)^2}\right),$$
$$B = \left(\frac{x_+}{(q_+-q)^2} + \frac{x_-}{(q_--q)^2}\right)(s\alpha[\mathbf{q}_-\mathbf{q}_+]_z - s\alpha_-[\mathbf{q}\mathbf{q}_+]_z + s\alpha_+[\mathbf{q}\mathbf{q}_-]_z) + 2[\mathbf{q}_-\mathbf{q}_+]_z - [\mathbf{Q}\mathbf{q}]_z,$$
$$s\alpha_\pm = \frac{\mathbf{q}_\pm^2}{x_\pm}, \quad s\alpha = \frac{\mathbf{k}_1^2}{x_1} + s\alpha_+ + s\alpha_- . \tag{47}$$

We can see that Bose symmetry is restored.

5 Subprocesses $e\gamma^* \to e\gamma; e+\gamma+\gamma$

Consider first the Compton subprocess[2]

$$\gamma^*(q) + e(p,\lambda_1) \to \gamma(k,\lambda) + e(p',\lambda_1).$$

For the chiral matrix elements we have (we chose $\lambda_1 = +1$)

$$m_\lambda^+ = \frac{N}{s}\bar{u}(p')\left[-\hat{p}\omega_\lambda(\hat{p}'+\hat{k})\hat{p}_2 - \hat{p}_2(\hat{p}-\hat{k})\hat{p}'\omega_{-\lambda}\right]\omega_+ u(p),$$
$$m_+^+ = -\frac{N}{s}\bar{u}(p')\hat{p}\hat{q}\hat{p}_2\omega_+ u(p),$$
$$m_-^+ = -\frac{N}{s}\bar{u}(p')\hat{p}_2\hat{q}\hat{p}'\omega_+ u(p). \tag{48}$$

The sum of the moduli square of the matrix elements is

$$T_e = \sum_\lambda |m_\lambda^+|^2 = 2\frac{\mathbf{q}^2}{\kappa\kappa'}[1+(1-x)^2], \tag{49}$$

with

$$\kappa = 2kp = \frac{\mathbf{k}^2}{x}, \quad \kappa' = 2kp' = \frac{1}{x(1-x)}(\mathbf{p}'x - \mathbf{k}(1-x))^2, \tag{50}$$

and $x = 2kp_2/2p_1p_2$ and $1-x$ are the energy fractions of photon and electron in the final state.

Consider now the double Compton subprocess (see Fig. 3a)

$$e(p,\eta) + \gamma^*(q) \to e(p',\eta) + \gamma(k_1,\lambda_1) + \gamma(k_2,\lambda_2). \tag{51}$$

The chiral matrix elements $m^\eta_{\lambda_1\lambda_2}$ are

$$m_{++}^+ = (m_{--}^-)^* = -\frac{s_1N_1N_2}{s}\bar{u}(p')\hat{p}\hat{q}\hat{p}_2\omega_+u(p), \tag{52}$$
$$m_{--}^+ = (m_{++}^-)^* = -\frac{s_1N_1N_2}{s}\bar{u}(p')\hat{p}_2\hat{q}\hat{p}'\omega_+u(p),$$
$$m_{+-}^+ = (m_{-+}^-)^* = \frac{N_1N_2}{s}\bar{u}(p')A_{+-}^+\omega_+u(p),$$
$$m_{-+}^+ = (m_{+-}^-)^* = \frac{N_1N_2}{s}\bar{u}(p')A_{-+}^+\omega_+u(p),$$

with $A_{-+}^+(k_1,k_2) = A_{+-}^+(k_2,k_1)$ and

$$A_{+-}^+(k_1,k_2) = \frac{s_1}{(p'-q)^2}\hat{p}_2(\hat{p}'-\hat{q})\hat{k}_1\hat{p}'\hat{k}_2 + \hat{p}(\hat{p}'+\hat{k}_1)\hat{p}_2(\hat{p}-\hat{k}_2)\hat{p}' + \frac{s_1}{(p+q)^2}\hat{k}_1\hat{p}\hat{k}_2(\hat{p}+\hat{q})\hat{p}_2, \tag{53}$$

with

$$s_1 = 2pp', \quad N_i^2 = \frac{2}{s_1\kappa_i\kappa_i'}, \quad \kappa_i = 2pk_i, \quad \kappa_i' = 2p'k_i. \tag{54}$$

To see the gauge invariance property of the two last amplitudes, we make the substitution $p_2 = (q-q_\perp)/\alpha_q$ in the second term of the r.h.s. and arrive at the form

$$A_{+-}^+(k_1,k_2) = ss_1\kappa_1'\left(\frac{x'}{(p'-q)^2} + \frac{1}{s\alpha_q}\right)\hat{k}_2 + ss_1\kappa_2\left(\frac{1}{(p+q)^2} - \frac{1}{s\alpha_q}\right)\hat{k}_1 + \frac{s_1}{(p+q)^2}\hat{k}_1\hat{p}\hat{k}_2\hat{q}_\perp\hat{p}_2 - \frac{s_1}{(p'-q)^2}\hat{p}_2\hat{q}_\perp\hat{k}_1\hat{p}'\hat{k}_2 - \hat{p}(\hat{p}'+\hat{k}_1)\hat{q}_\perp(\hat{p}-\hat{k}_2)\hat{p}'\frac{s}{s\alpha_q}. \tag{55}$$

We can verify that this expression turns to zero at $\mathbf{q} = 0$. In fact, we can use

$$(p'-q)^2 = -\mathbf{q}^2 + 2\mathbf{p}'\mathbf{q} - sx'\alpha_q,$$
$$(p+q)^2 = -\mathbf{q}^2 + s\alpha_q, \alpha_q = \alpha' + \alpha_1 + \alpha_2,$$
$$x' + x_1 + x_2 = 1, \quad s\alpha' = \frac{(s\mathbf{p}')^2}{x'}, \quad s\alpha_i = \frac{\mathbf{k}_i^2}{x_i},$$
$$\kappa_i = s\alpha_i, \quad \kappa_i' = \frac{1}{x'x_i}(\mathbf{k}_ix' - \mathbf{p}'x_i)^2. \tag{56}$$

The further strategy is similar to the one mentioned above (43).

6 Subprocesses $e\gamma^* \to e\pi^+\pi^-, e\mu^+\mu^-$

The matrix element of the pion pair production subprocess

$$e(p,\eta) + \gamma^*(q) \to \pi^+(q_+) + \pi^-(q_-) + e(p',\eta)$$

can be written in the form

$$m^\eta = \bar{u}(p')[\hat{B} + \hat{D}]\omega_\eta u(p), \tag{57}$$

where the bremsstrahlung mechanism contribution is (see Fig. 3b)

$$\hat{B} = \frac{1}{q_1^2}\left[B\hat{q}_1 + \frac{1}{s(p+q)^2}\hat{q}_1\hat{q}\hat{p}_2 - \frac{1}{s(p'-q)^2}\hat{p}_2\hat{q}\hat{q}_1\right],$$
$$q_1 = q_+ + q_-, \quad q_2 = p' - p_1,$$
$$\hat{D} = \frac{1}{q_2^2}\left[D(2\hat{q}_- + \hat{q}_2) - 2\frac{x_-}{(q-q_-)^2}\hat{q}_\perp + \frac{2(\mathbf{q}^2 - 2\mathbf{q}\mathbf{q}_-)}{s(q-q_-)^2}\hat{p}_2\right]. \tag{58}$$

[2] The case of real initial photons was considered in [10].

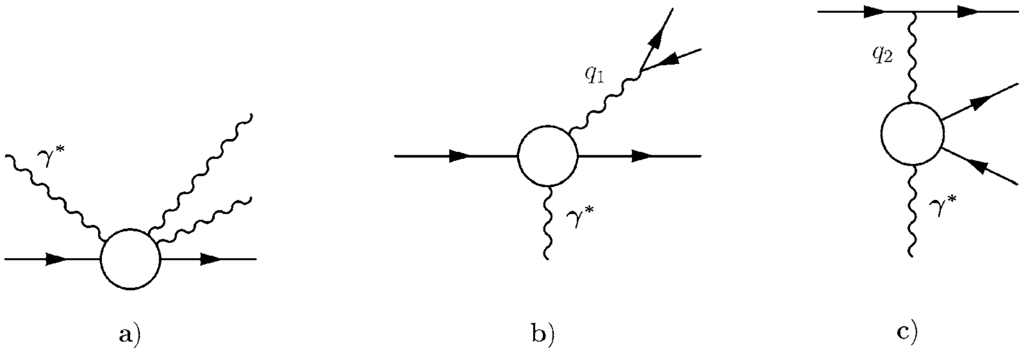

Fig. 3. Feynman diagrams describing **a** the subprocess $\gamma^* e^- \to \gamma\gamma e^-$ and the pair production $\gamma^* e \to e a\bar{a}$ subprocess by the bremsstrahlung **b** and double photon **c** mechanisms

For the squares of the moduli of the chiral amplitudes, which enter in (23) and (24), we have

$$T_3^{(\pi)} = |m^+|^2 = \mathrm{Sp}\left(\hat{p}'(\hat{B}+\hat{D})\hat{p}(\tilde{B}+\tilde{D})\omega_+\right), \quad (59)$$

with B and D specified below (62).

For the subprocess of the muon pair production,

$$e(p,\eta) + \gamma^*(q) \to \mu^+(q_+) + \mu^-(q_-) + e(p',\eta), \quad (60)$$

bremsstrahlung and the two-photon mechanisms must be taken into account (see Fig. 3b,c):

$$m_\lambda^+ = \frac{1}{q_1^2}\bar{u}(p')B_\mu\omega_+ u(p)\bar{u}(q_-)\gamma^\mu\omega_\lambda v(q_+) + \frac{1}{q_2^2}\bar{u}(p')\gamma_\nu\omega_+ u(p)\bar{u}(q_-)D_\nu\omega_\lambda v(q_+), \quad (61)$$

with the double photon mechanism contribution (not considered in [5])

$$D_\nu = D\gamma_\nu + \frac{1}{s(q-q_+)^2}\gamma_\nu\hat{q}\hat{p}_2 - \frac{1}{s(q-q_-)^2}\hat{p}_2\hat{q}\gamma_\nu,$$

and the bremsstrahlung mechanism one

$$B_\mu = B\gamma_\mu - \frac{1}{s(p'-q)^2}\hat{p}_2\hat{q}\gamma_\mu + \frac{1}{s(p+q)^2}\gamma_\mu\hat{q}\hat{p}_2,$$

with

$$B = \frac{x'}{(p'-q)^2} + \frac{1}{(p+q)^2}, \quad D = \frac{x_-}{(q_- - q)^2} - \frac{x_+}{(q-q_+)^2},$$
$$x_\pm = \frac{2p_2q_\pm}{s}, \quad x' = \frac{2p_2p'}{s}, \quad x_+ + x_- + x' = 1.$$

To perform the conversion in the Lorentz indices μ,ν in (66), one can use the projection operators. In the case of equal chiralities $\eta = \lambda = +1$ we choose the projection operator as

$$P_+ = \frac{\bar{u}(p)\hat{q}_+\omega_+ u(q_-)}{\bar{u}(p)\hat{q}_+\omega_+ u(q_-)}. \quad (62)$$

Inserting it and using the relation $\omega_+ u(p)\bar{u}(p) = \omega_+\hat{p}$, we obtain

$$m_+^+ = \frac{-2}{\bar{u}(p)\hat{q}_+\omega_+ u(q_-)}\bar{u}(p')\Bigg[\left(\frac{D}{q_2^2}+\frac{B}{q_1^2}\right)\hat{q}_-\hat{q}_+\hat{p} + \frac{\hat{q}_-\hat{q}_+\hat{p}\hat{q}_\perp\hat{p}_2}{s}\left(\frac{1}{q_2^2(q-q_+)^2} - \frac{1}{q_1^2(p+q)^2}\right) + \frac{\hat{p}_2\hat{q}_\perp\hat{q}_-\hat{q}_+\hat{p}}{s}\left(\frac{1}{q_2^2(q_- - q)^2} - \frac{1}{q_1^2(p'-q)^2}\right)\Bigg] \times\omega_+ v(q_+) = \frac{-2}{\bar{u}(p)\hat{q}_+\omega_+ u(q)_-}\bar{u}(p')A_+^+\omega_+ v(q_+). \quad (63)$$

In the case of opposite chiralities $\eta = -\lambda = +1$ we use the projection operator

$$P_- = \frac{\bar{u}(p)\omega_- u(q_-)}{\bar{u}(p)\omega_- u(q_-)}. \quad (64)$$

Similar calculations lead to the result

$$m_-^+ = \frac{2}{\bar{u}(p)\omega_- u(q_-)}\bar{u}(p')\Bigg[\left(\frac{D}{q_2^2}+\frac{B}{q_1^2}\right)2(pq_-) + 2\frac{\hat{p}\hat{q}_-\hat{q}_\perp\hat{p}_2}{s}\left(\frac{1}{q_2^2(q-q_+)^2} + \frac{1}{q_1^2(p_1-q_-)^2}\right) - \frac{\hat{p}\hat{q}_\perp\hat{p}_2\hat{q}_-}{s}\left(\frac{1}{q_2^2(q-q_-)^2} + \frac{1}{q_1^2(p+q)^2}\right) - \frac{\hat{q}_-\hat{p}_2\hat{q}_\perp\hat{p}}{s}\left(\frac{1}{q_2^2(q-q_-)^2} + \frac{1}{q_1^2(p+q)^2}\right)\Bigg]\omega_- v(q_+) = \frac{2}{\bar{u}(p)\omega_- u(q_-)}\bar{u}(p')A_-^+\omega_- v(q_+). \quad (65)$$

The property of A_+^+, A_-^+ tending to zero as $|\mathbf{q}| \to 0$ is explicitly seen from (71) and (72).

For the sum of the squares of the chiral amplitudes entering into (23) and (24), one has

$$T_3^{(\mu)} = \sum|m_\lambda^+|^2 = \frac{1}{(pq_+)(q-q_+)}\mathrm{Sp}(\hat{p}'A_+^+\hat{q}_+\tilde{A}_+^+\omega_+) + \frac{2}{pq_-}\mathrm{Sp}(\hat{p}'A_-^+\hat{q}_+\tilde{A}_-^+\omega_+). \quad (66)$$

The further strategy is straightforward.

7 Subprocess $e\gamma^* \to ee\bar{e}$

The kinematics of the subprocess is defined as

$$e(p,l_p)+\gamma^*(q)\to e(p_1,l_1)+e(p_2,l_2)+\bar{e}(p_+,t)\,,$$

with $l_i, t=\pm$ the chiralities of the initial and final fermions. Without loss of generality we can put $l_p=+$ below. For the sum of the chiral states of the moduli square of the relevant matrix element we obtain

$$\sum\left|M^{l_p}_{l_1l_2t}\right|^2=2\left[\left|M^+_{++-}\right|^2+\left|M^+_{+-+}\right|^2+\left|M^+_{-++}\right|^2\right]\,. \tag{67}$$

Eight Feynman diagrams are relevant, which form four gauge invariant sets of amplitudes:

$$\begin{aligned}M^+_{l_1l_2t}&=(4\pi\alpha)^{\frac{3}{2}}\left(-\frac{1}{s_1}\right)\\ &\times\big(\delta_{l_1,+}\delta_{t,-l_2}\left[\bar{u}^{l_2}(p_2)\gamma_\lambda v^t(p_+)\bar{u}^{l_1}A_\lambda u^+(p)\right.\\ &\left.+\bar{u}^{l_1}(p_1)\gamma_\sigma\bar{u}^+(p)\bar{u}^{l_2}B_\sigma v^t(p_+)\right]\\ &+\delta_{l_2,+}\delta_{t,-l_1}\left[\bar{u}^{l_1}(p_1)\gamma_\eta v^t(p_+)\bar{u}^{l_2}(p_2)D_\eta u^+(p)\right.\\ &\left.+\bar{u}^{l_2}(p_2)\gamma_\delta\bar{u}^+(p)\bar{u}^{l_1}(p1)C_\delta v^t(p_+)\right]\big)\,.\end{aligned} \tag{68}$$

Applying projection operators to provide the conversion on the vector indices we have

$$\begin{aligned}|M^+_{++-}|^2&=\frac{(4\pi\alpha)^3}{2s_1^2pp_+}\left[\frac{1}{p_2p_+}\frac{1}{4}\mathrm{Sp}\hat{p}_1m^{(1)}_{++-}\hat{p}_+(m^{(1)}_{++-})^+\right.\\ &+\frac{1}{p_+p_1}\frac{1}{4}\mathrm{Sp}\hat{p}_2m^{(2)}_{++-}\hat{p}_+(m^{(2)}_{++-})^+\frac{1}{p_1p_+p_2p_+}\\ &\left.\times\frac{1}{4}\mathrm{Sp}\hat{p}_1m^{(1)}_{++-}\hat{p}_+(m^{(2)}_{++-})^+\hat{p}_2\hat{p}_+\right],\end{aligned} \tag{69}$$

$$|M^+_{+-+}|^2=\frac{(4\pi\alpha)^3}{2s_1^2pp_2}\frac{1}{4}\mathrm{Sp}\hat{p}_1m_{+-+}\hat{p}_+(m_{+-+})^+,$$

$$|M^+_{-++}|^2=\frac{(4\pi\alpha)^3}{2s_1^2pp_1}\frac{1}{4}\mathrm{Sp}\hat{p}_2m_{-++}\hat{p}_+(m_{-++})^+,$$

with

$$\begin{aligned}m_{+-+}&=\gamma_\sigma\hat{p}\hat{p}_2B_\sigma+A_\lambda\hat{p}\hat{p}_2\gamma_\lambda,\\ m_{-++}&=\gamma_\delta\hat{p}\hat{p}_1C_\delta+D_\eta\hat{p}\hat{p}_1\gamma_\eta,\\ m^{(1)}_{++-}&=A_\lambda\hat{p}\hat{p}_+\hat{p}_2\gamma_\lambda+\gamma_\sigma\hat{p}\hat{p}_+\hat{p}_2B_\sigma,\\ m^{(2)}_{++-}&=\gamma_\sigma\hat{p}\hat{p}_+\hat{p}_1C_\sigma+D_\eta\hat{p}\hat{p}_+\hat{p}_1\gamma_\eta,\end{aligned} \tag{70}$$

$$\begin{aligned}A_\lambda&=\frac{\hat{q}_\perp(\hat{p}_1-\hat{q})\gamma_\lambda}{(p_1-q)^2}+\frac{\gamma_\lambda(\hat{p}+\hat{q})\hat{q}_\perp}{(p+q)^2},\\ B_\sigma&=\frac{\hat{q}_\perp(\hat{p}_2-\hat{q})\gamma_\sigma}{(p_2-q)^2}+\frac{\gamma_\sigma(\hat{q}-\hat{p}_+)\hat{q}_\perp}{(p_+-q)^2},\\ C_\sigma&=\frac{\hat{q}_\perp(\hat{p}_1-\hat{q})\gamma_\sigma}{(p_1-q)^2}+\frac{\gamma_\sigma(\hat{q}-\hat{p}_+)\hat{q}_\perp}{(q-p_+)^2},\\ D_\eta&=\frac{\hat{q}_\perp(\hat{p}_2-\hat{q})\gamma_\eta}{(p_2-q)^2}+\frac{\gamma_\eta(\hat{p}+\hat{q})\hat{q}_\perp}{(p+q)^2}\,.\end{aligned} \tag{71}$$

8 Conclusion

In [9], we wrote down the explicit expressions for the spin matrix elements $\mathcal{M}_{ij}$ for the subprocesses of the type $2\to 2$ that are reviewed here. For the subprocesses of the type $2\to 3$ we formulated the algorithm of calculation of the spin matrix elements. We considered all possibilities of pair creation in the mentioned subprocesses, as these were not completely considered in a recent work [5]. The gauge condition $\mathcal{M}_{ij}(q)\to 0$ for $|\mathbf{q}|\to 0$ is explicitly fulfilled in all cases. The subprocesses with the pions in the final state were also considered in the paper for the first time.

Radiative corrections to the chiral amplitude were calculated only for some subprocesses of type $2\to 2$ [13].

The magnitude of the cross sections (21)–(23) is of the order of $\alpha^n/\mu^2\gg\alpha^n/s$, $n=4,5,6$, where $\mu^2=\max(s_1,s_2)$ is large enough to be measured and does not depend on s. The strategy of the calculation of the cross section, using the helicity amplitudes of the subprocesses $2\to 3$, is described above and can be implemented into numerical programs that take into account details of the experiments.

Acknowledgements. This work was supported by the Byelorussian Foundation for Basic Research, grant No. F07D-005.

References

1. Zeroth-Order Design Report for the Next Linear Collider, Report No. SLAC-474 (1996)
2. JLC Design Study, KEK Report 97-1 (1997)
3. R.D. Heuer et al., Technical Design Report of 500 GeV Electron Positron Collider with Integrated X-Ray Facility (DESY 2001-011, March 2001)
4. Physics Potential and Development of $\mu^+\mu^-$ Colliders, ed. by D. Cline, AIP Conference Proceedings 441 (1997)
5. C. Carimalo, A. Schiller, V.G. Serbo, Eur. Phys. J. C **23**, 633 (2002)
6. V.N. Baier, V.S. Fadin, V.A. Khoze, E.A. Kuraev, Phys. Rep. **78**, 293 (1982)
7. V. Berestetsky, E. Lifshits, L. Pitaevski, Quantum Electrodynamics (Nauka, Moscow, 1989), p. 43
8. F.A. Berends et. al, Nucl. Phys. B **206**, 53 (1982)
9. E. Bartoš et al., Nucl. Phys. B **676**, 481 (2004)
10. E. Bartoš et al., Nucl. Phys. B **676**, 390 (2004)
11. V.I. Borodulin, R.N. Rogalev, S.R. Slabospitsky (Serpukhov, IHEP), CORE: Compendium of relations: Version 2.1. Preprint IFVE-95-90, IHEP-95-90, Jul 1995. 108pp. e-Print Archive: hep-ph/9507456
12. Private communication of S. Sikach (Minsk, Institute of Physics BAS, Belarus)
13. V. Bytev, E. Bartos, M. Galynskii, E. Kuraev, JETP **130**, 259 (2006)